Sample Question Paper - 24
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Find the n^{th} term of the following A.P. :

$3\sqrt{2}, 4\sqrt{2}+1, 5\sqrt{2}+2, 6\sqrt{2}+3, \ldots$

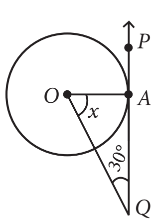
2. In a certain distribution, mean and median are 9.5 and 10 respectively. Find the mode of the distribution, using an empirical relation.

3. Prove that the roots of quadratic equation $21x^2 - 2x + 1/21 = 0$ are real and repeated.

OR

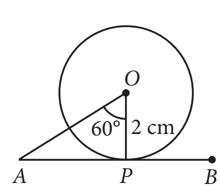
Find the roots of the quadratic equation $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$.

4. From the given figure, find x. Also find the length of AQ if radius of the circle is 6 cm.



OR

In the adjoining figure, AB is the tangent to the circle with centre O at P. If $\angle AOP = 60°$ and radius is 2 cm, then find AP.



5. Find the number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm.

6. If $\sum f_i = 11$, $\sum f_i x_i = 2p + 52$ and the mean of distribution is 6, then find the value of p.

SECTION - B

7. A pole of height 5 m is fixed on the top of a tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 30°. Find the distance of the foot of the tower from point A. [Take $\sqrt{3} = 1.732$]

8. Find the 37th term of the A.P. $\sqrt{x}, 3\sqrt{x}, 5\sqrt{x}, \ldots\ldots$.

OR

How many numbers lie between 10 and 201, which when divided by 3 leave a remainder 2?

9. Draw a tangent to the circle of radius 1.8 cm at the point P, without using its centre.

10. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, then find the two numbers.

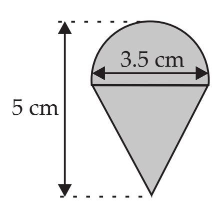
SECTION - C

11. The shadow of a tower standing on a leveled ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

OR

The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point R, 40 m vertically above X, the angle of elevation is 45°. Find the height of the tower PQ.

12. A spinning top (lattu) is shaped like a cone surmounted by a hemisphere (see figure). The entire spinning top is 5 cm in height and the diameter of the spinning top is 3.5 cm. Find the total surface area of the spinning top. $\left(\text{Take } \pi = \frac{22}{7} \text{ and } \sqrt{13.625} = 3.7\right)$



Case Study - 1

13. Suppose you are interested in analysing the monthly groceries expenditure of a family. The data of monthly grocery expenditure of 200 families is given in the following table.

Monthly expenditure (in ₹)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000
Number of families	28	46	54	42	30

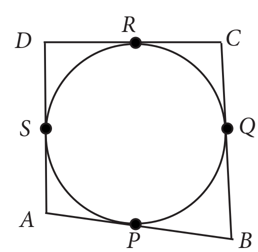
Based on the above information, answer the following questions.

(i) Find the median of the monthly expenditure.

(ii) Find the sum of upper limit and lower limit of modal class.

Case Study - 2

14. In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) If O is the centre of the circular fountain, then find $\angle OSA$.

(ii) If $DR = 7$ cm and $AD = 11$ cm, then find AP.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Given A.P. is $3\sqrt{2}, 4\sqrt{2}+1, 5\sqrt{2}+2, 6\sqrt{2}+3, \ldots$

Here, first term $a = 3\sqrt{2}$ and

Common difference, $d = 4\sqrt{2}+1-3\sqrt{2} = \sqrt{2}+1$

$\because \; a_n = a+(n-1)d = 3\sqrt{2}+(n-1)(\sqrt{2}+1)$

$= 3\sqrt{2}+n\sqrt{2}-\sqrt{2}+n-1$

$= (n+2)\sqrt{2}+(n-1)$, is the required n^{th} term.

2. We know that, empirical relation between mean, median and mode is

Mode = 3 Median −2 Mean ...(i)

From given, we have, Mean = 9.5, Median = 10

$\therefore$ Mode = 3(10) – 2(9.5) (Using (i))

$\Rightarrow$ Mode = 11

3. We have, $21x^2 - 2x + 1/21 = 0$

$\Rightarrow 441x^2 - 42x + 1 = 0$

Here, $a = 441$, $b = -42$ and $c = 1$.

$\therefore \; D = b^2 - 4ac = (-42)^2 - 4(441)(1) = 1764 - 1764 = 0$

Hence, both roots are real and repeated.

OR

Given, $a^2x^2 - (a^2b^2 + 1)x + b^2 = 0$

$\Rightarrow a^2x^2 - a^2b^2x - x + b^2 = 0 \Rightarrow a^2x(x-b^2) - 1(x-b^2) = 0$

$\Rightarrow (a^2x - 1)(x - b^2) = 0$

$\Rightarrow a^2x - 1 = 0$ or $x - b^2 = 0 \Rightarrow x = 1/a^2$ or $x = b^2$

$\therefore \; 1/a^2, b^2$ are the required roots.

4. Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \; OA \perp PQ \Rightarrow \angle OAQ = 90°$

$\therefore$ In ΔOAQ, $x + 30° + 90° = 180°$

[By angle sum property]

$\Rightarrow x = 180° - 90° - 30° = 60°$

Also, $\tan 30° = \dfrac{OA}{AQ} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{6}{AQ}$ [$\because$ Radius, $OA = 6$ cm]

$\Rightarrow AQ = 6\sqrt{3}$ cm

OR

Given, $\angle AOP = 60°$ and $OP = 2$ cm

Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \; OP \perp AB$

$\Rightarrow \angle OPA = 90°$

$\Rightarrow \Delta OAP$ is right angle triangle.

In ΔOPA,

$\tan 60° = \dfrac{AP}{OP} \Rightarrow \sqrt{3} = \dfrac{AP}{2} \Rightarrow AP = 2\sqrt{3}$ cm

5. Let n be the number of solid spheres formed by melting the solid metallic cylinder

$\therefore \; n \times$ Volume of one solid sphere

= Volume of the solid cylinder

$\Rightarrow n \times \dfrac{4}{3}\pi R^3 = \pi(r)^2 \times h$

(where R, r be the radius of sphere and cylinder respectively and h be height of cylinder)

$\Rightarrow n \times \dfrac{4}{3}(3)^3 = (2)^2 \times 45$

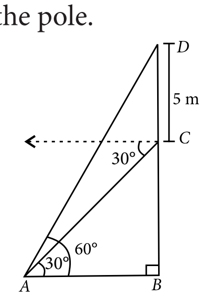
$\Rightarrow n \times \dfrac{4}{3} \times 27 = 180 \Rightarrow n = \dfrac{180 \times 3}{4 \times 27} = 5$

Thus, the number of solid spheres that can be formed is 5.

6. Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} \Rightarrow 6 = \dfrac{2p+52}{11}$

$\Rightarrow 66 = 2p + 52 \Rightarrow 2p = 14 \Rightarrow p = 7$

7. Let BC be the tower and CD be the pole.



In ΔABC, $\dfrac{BC}{AB} = \tan 30°$

$\Rightarrow \dfrac{BC}{AB} = \dfrac{1}{\sqrt{3}} \Rightarrow AB = BC\sqrt{3}$ ...(i)

In ΔABD, $\dfrac{BD}{AB} = \tan 60°$

$\Rightarrow \dfrac{BC+CD}{AB} = \sqrt{3} \Rightarrow \dfrac{BC+5}{BC\sqrt{3}} = \sqrt{3}$ [Using (i)]

$\Rightarrow BC + 5 = 3BC \Rightarrow 2BC = 5 \Rightarrow BC = 2.5$ m

$\therefore$ Distance of foot of tower from point $A = AB$

$= BC\sqrt{3} = 2.5 \times 1.732 = 4.33$ m.

8. We have, $a = \sqrt{x}, d = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$

Now, $a_n = a + (n-1)d$

$\therefore \; a_{37} = a + 36d = \sqrt{x} + 36(2\sqrt{x})$

$\Rightarrow a_{37} = \sqrt{x} + 72\sqrt{x} = (1+72)\sqrt{x} = 73\sqrt{x}$

OR

The required numbers are 11, 14, 17, ..., 200.

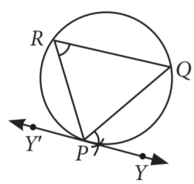
This is an A.P. in which $a = 11$, $d = 14 - 11 = 3$

Now, $a_n = 200 \Rightarrow a + (n-1)d = 200$

$\Rightarrow\ 11 + (n-1) \times 3 = 200 \ \Rightarrow\ 3(n-1) = 189$

$\Rightarrow\ (n-1) = 63 \ \Rightarrow\ n = 64$

9. Steps of Construction :



Step-I : Draw a circle of radius 1.8 cm and take a point P on the circle.

Step-II : Draw a chord PQ through the point P on the circle.

Step-III : Take any point R in the major arc and join PR and RQ.

Step-IV : On taking PQ as base, construct $\angle QPY = \angle PRQ$.

Step-V : Produce YP to Y'. Then, $Y'PY$ is the required tangent at point P.

10. Let the smaller number be x.

$\therefore$ Larger number is $x + 4$.

According to question, $\dfrac{1}{x} - \dfrac{1}{x+4} = \dfrac{4}{21}$

$\Rightarrow \dfrac{x+4-x}{x(x+4)} = \dfrac{4}{21} \Rightarrow \dfrac{1}{x(x+4)} = \dfrac{1}{21}$

$\Rightarrow x^2 + 4x - 21 = 0 \Rightarrow x^2 + 7x - 3x - 21 = 0$

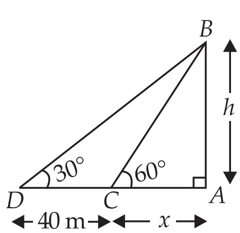
$\Rightarrow (x+7)(x-3) = 0 \Rightarrow x = 3$ or $x = -7$

If $x = 3$, then $x + 4 = 3 + 4 = 7$

If $x = -7$, then $x + 4 = -7 + 4 = -3$

Therefore, the pairs of numbers are 3 and 7 or -7 and -3.

11. Let AB be the tower and AC & AD be its shadows when the angles of elevation are 60° and 30° respectively. Then $CD = 40$ metres. Let h be the height of the tower and let $AC = x$ metres.



In ΔABC, right angled at A, we have

$\tan 60° = \dfrac{AB}{AC} \Rightarrow \sqrt{3} = \dfrac{h}{x}$

$\Rightarrow\ h = \sqrt{3}x \ \Rightarrow\ x = \dfrac{h}{\sqrt{3}}$ ...(i)

In ΔDAB, we have

$\tan 30° = \dfrac{AB}{AD}$

$\Rightarrow\ \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+40} \ \Rightarrow\ x + 40 = \sqrt{3}h$ ...(ii)

Putting value of x from (i) in (ii), we get

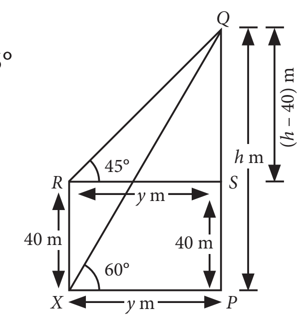
$\dfrac{h}{\sqrt{3}} + 40 = \sqrt{3}h \quad \Rightarrow\ h + 40\sqrt{3} = 3h$

$\Rightarrow 2h = 40\sqrt{3} \Rightarrow\ h = 20\sqrt{3}$

Thus, the height of the tower is $20\sqrt{3}$ metres.

OR

Let the height of tower PQ be h m and distance PX be y m



Given, $RX = 40$ m $= SP$,

$\angle QXP = 60°$ and $\angle QRS = 45°$

In ΔPXQ, $\tan 60° = \dfrac{PQ}{XP}$

$\Rightarrow\ \sqrt{3} = \dfrac{h}{y}$

$\Rightarrow\ y = \dfrac{h}{\sqrt{3}}$ ...(i)

In ΔRSQ, $\tan 45° = \dfrac{QS}{RS}$

$\Rightarrow\ 1 = \dfrac{PQ - SP}{XP}$ $[\because RS = XP]$

$\Rightarrow\ 1 = \dfrac{h-40}{y} \Rightarrow y = h - 40$ ...(ii)

From (i) and (ii), we get

$\dfrac{h}{\sqrt{3}} = h - 40 \ \Rightarrow\ \dfrac{h - \sqrt{3}h}{\sqrt{3}} = -40$

$\Rightarrow\ -h(\sqrt{3} - 1) = -40\sqrt{3}$

$\Rightarrow h = \dfrac{40\sqrt{3}}{\sqrt{3}-1} = \dfrac{40\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow\ h = 20(3 + \sqrt{3}) = 20 \times 4.732 = 94.64$ m

12. We have, radius of hemispherical part of the spinning top= radius of conical part $= r = \dfrac{3.5}{2}$ cm

Height of the conical part $(h) = 5 - \dfrac{3.5}{2} = 3.25$ cm

Slant height of the conical part $(l) = \sqrt{r^2 + h^2}$

$= \sqrt{\left(\dfrac{3.5}{2}\right)^2 + (3.25)^2} = 3.7$ cm

Total surface area of the spinning top = curved surface area of hemispherical part + curved surface area of conical part $= 2\pi r^2 + \pi r l = \pi r(2r + l)$

$$= \frac{22}{7} \times \frac{3.5}{2}\left(2 \times \frac{3.5}{2} + 3.7\right)$$

$$= \frac{22}{7} \times \frac{3.5}{2} \times 7.2 = 39.6 \text{ cm}^2.$$

13. (i) We have, the following table :

Class interval	**Frequency (f_i)**	**Cumulative frequency ($c.f.$)**
0-1000	28	28
1000-2000	46	74
2000-3000	54	128
3000-4000	42	170
4000-5000	30	200
	$\Sigma f_i = n = 200$	

Here, $\frac{n}{2} = \frac{200}{2} = 100$

$\therefore$ Median class = 2000 – 3000

$l = 2000$, $c.f. = 74$, $f = 54$, $h = 1000$

$$\therefore \quad \text{Median} = 2000 + \left(\frac{100 - 74}{54}\right) \times 1000$$

$$= 2000 + \frac{26000}{54} = 2000 + 481.481 = 2481.5$$

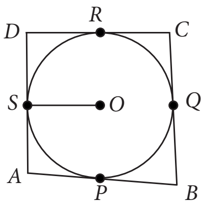
(ii) Since, maximum frequency is 54

$\therefore$ Modal class = 2000 – 3000

Hence, sum of lower and upper limit = 2000 + 3000

= 5000

14. (i)



Here, OS the is radius of circle.

Since radius at the point of contact is perpendicular to tangent.

So, $\angle OSA = 90°$

(ii) Since, the lengths of tangents drawn from an external point to a circle are equal.

$AP = AS = AD - DS = AD - DR$

$= 11 - 7 = 4$ cm